Chapter One

Real Number

Mathematics is originated from the process of expressing quantities in symbols or numbers. The history of numbers is as ancient as the history of human civilization. Greek Philosopher Aristotle According to the formal inauguration of mathematics occurs in the practice of mathematics by the sect of priest in ancient Egypt. So, the number based mathematics is the creation of about two thousand years before the birth of Christ. After that, moving from many nations and civilization, numbers and principles of numbers have gained an universal form at present.

The mathematicians in India first introduce zero (0) and 10 based place value system for counting natural numbers, which is considered a milestone in describing numbers. Chinese and Indian mathematicians extended the idea zero, real numbers, negative number, integer and fractional numbers which the Arabian mathematicians accepted in the middle age. But the credit of expressing number through decimal fraction is awarded to the Muslim Mathematicians. Again they introduce first the irrational numbers in square root form as a solution of the quadratic equation in algebra in the 11th century. According to the historians, very near to 50 BC the Greek Philosophers also felt the necessity of irrational number for drawing geometric figures, especially for the square root of 2. In the 19th century European Mathematicians gave the real numbers a complete shape by systematization. For daily necessity, a student must have a vivid knowledge about 'Real Numbers'. In this chapter real numbers are discussed in detail.

At the end of this chapter, the students will be able to –

- Classify real numbers
- Express real numbers into decimal and determine approximate value
- Explain the classification of decimal fractions
- Explain recurring decimal numbers and express fractions into recurring decimal numbers
- Transform recurring decimal fraction into simple fractions
- Explain non-terminating non-recurring decimal fraction
- Explain non-similar and similar decimal fraction
- Add, subtract multiply and divide the recuring decimal fraction and solve various problems related to them.

Natural Number

$1, 2, 3, 4.........$ etc. numbers are called natural number or positive whole numbers. $2, 3, 5, 7.........$ etc. are prime numbers and $4, 6, 8, 9,.........$ etc. are composite numbers.

Integers

All numbers (both positive and negative) with zero (0) are called integers i.e. $.......-3, -2, -1, 0, 1, 2, 3.........$ etc. are integers.

Fractional Number

If p, q are co-prime numbers ; $q \neq 0$ and $q \neq 1$, numbers expressed in $\frac{p}{q}$ form are called fractional number.

Example : $\frac{1}{2}, \frac{3}{2}, \frac{-5}{3}$ etc. are fractional numbers.

If $p < q$, then it is a proper fraction and if $p > q$ then it is an improper fraction : Example $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, ...$ etc. proper and $\frac{3}{2}, \frac{4}{3}, \frac{5}{3}, \frac{5}{4},....$ etc. improper fraction.

Rational Number

If p and q are integers and $q \neq 0$, number expressed in the form $\frac{p}{q}$ is called rational number. For example : $\frac{3}{1} = 3, \frac{11}{2} = 5.5, \frac{5}{3} = 1.666...$ etc. are rational numbers. Rational numbers can be expressed as the ratio of two integers. So, all integers and all fractional numbers are rational numbers.

Irrational Number

Numbers which cannot be expressed in $\frac{p}{q}$ form, where p, q are integers and $q \neq 0$ are called Irrational Numbers. Square root of a number which is not perfect square, is an irrational number. For example: $\sqrt{2} = 1.414213.....$, $\sqrt{3} = 1.732.....$, $\frac{\sqrt{5}}{2} = 1.58113....$ etc. are irrational numbers. Irrational number cannot be expressed as the ratio of two integers.

Decimal Fractional Number

If rational and irrational numbers are expressed in decimal, they are known as decimal fractional numbers. As for instance, $3 = 3 \cdot 0, \frac{5}{2} = 2 \cdot 5, \frac{10}{3} = 3 \cdot 3333.......$, $\sqrt{3} = 1 \cdot 732.........$etc. are decimal fractional numbers. After the decimal, if the number of digits are finite, it is terminating decimals and if it is infinite it is known as non-terminating decimal number. For example, 0.52, 3.4152 etc. are terminating decimals and $1 \cdot 333.......$, $2 \cdot 123512367...........$ etc. are non-terminating decimals. Again, if the digits

after the decimal of numbers are repeated among themselves, they are known recurring decimals and if they are not repeated, they are called non-recurring decimals. For example : $1\cdot2323........$, $5\cdot\dot{6}\dot{5}\dot{4}$ etc. are the the recurring decimals and $0\cdot523050056........$, $2\cdot12340314........$etc. are non-recurring decimals.

Real Number

All rational and irrational numbers are known as real numbers. For example :

$$0, \pm1, \pm2, \pm3,.......... \quad \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{4}{3},........ \quad \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}......$$

$1\cdot23,\ 0\cdot415,\ 1\cdot3333.......,\ 0\cdot\dot{6}\dot{2},\ 4\cdot120345061..........$ etc. are real numbers.

Positive Number

All real numbers greater than zero are called positive numbers. As for instance

$1,\ 2,\ \frac{1}{2},\ \frac{3}{2},\ \sqrt{2},\ 0\cdot415,\ 0\cdot\dot{6}\dot{2},\ 4\cdot120345061.............$ etc. are positive numbers.

Negative Number

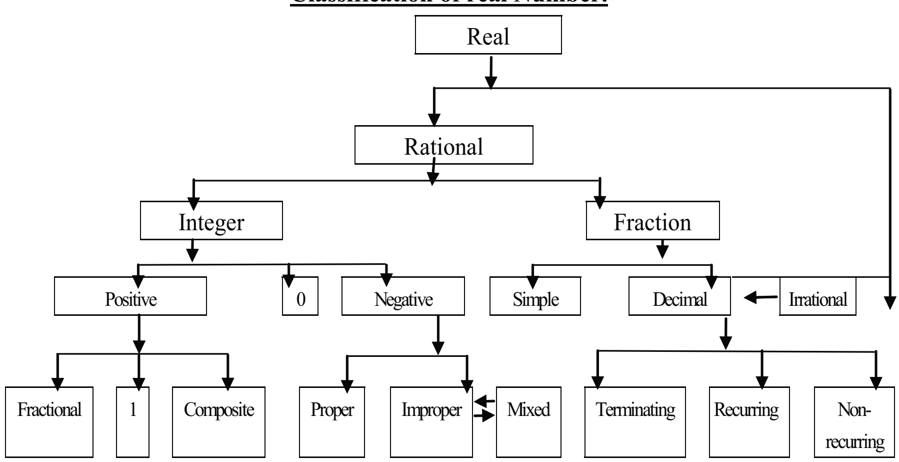
All real numbers less than zero are called negative numbers. For example, $-1,\ -2,\ -\frac{1}{2},\ -\frac{3}{2},\ -\sqrt{2},\ -0\cdot415,\ -0\cdot\dot{6}\dot{2},\ -4\cdot120345061.............$ etc. are negative numbers.

Non-Negative Number

All positive numbers including zero are called non-negative numbers. For example,

$0,\ 3,\ \frac{1}{2},\ \ 0\cdot612,\ 1\cdot\dot{3},\ 2\cdot120345.............$ etc. are non-negative numbers.

<u>Classification of real Number.</u>



Activity : Show the position of the numbers $\frac{3}{4}$, 5, -7, $\sqrt{13}$, 0, 1, $\frac{9}{7}$, 12, $2\frac{4}{5}$, $1\cdot1234......$, $.3\dot{2}\dot{3}$ in the classification of real numbers.

Example 1. Determine the two irrational numbers between $\sqrt{3}$ and 4.

Solution : Here, $\sqrt{3} = 1.7320508......$

Let, $a = 2.030033000333.....$

and $b = 2.505500555.......$

Clearly :both a and b are real numbers and both are greater than $\sqrt{3}$ and less than 4.

i.e., $\sqrt{3} < 2.03003300333......... < 4$

and $\sqrt{3} < 2.505500555............... < 4$

Again, a and b cannot be expressed into fractions.

$\therefore$ a and b are the two required irrational numbers.

Basic characteristics of addition and multiplication over a real number :

1. If a, b are real numbers, $(i)\ a+b$ is real and $(ii)\ ab$ is a real number
2. If a, b are real numbers, $(i)\ a+b=b+a$ and $(ii)\ ab=ba$
3. If a, b, c are real numbers, $(i)\ (a+b)+c=a+(b+c)$ and $(ii)\ (ab)c=a(bc)$
4. If a is a real number, in real numbers there exist only two number 0 and 1 where $(i)\ 0 \neq 1$ $(ii)\ a+0=a$ $(iii)\ a.1=1.a=a$
5. If a is a real number, $(i)\ a+(-a)=0$ (ii) If $a \neq 0$, $a.\frac{1}{a}=1$
6. If a, b, c are real numbers, $a(b+c)=ab+ac$
7. If a, b are real numbers, $a<b$ or $a=b$ or $a>b$
8. If a, b, c are real numbers and $a<b$, $a+c<b+c$
9 If a, b, c are real numbers and $a<b$, $(i)\ ac<bc$ where $c<0$
(ii) If $ac>bc$, $c<0$

Proposition : $\sqrt{2}$ is an irrational number.

We know,

$1<2<4$

$\therefore\ \sqrt{1}<\sqrt{2}<\sqrt{4}$

or, $1<\sqrt{2}<2$

Proof : $1^2=1$, $\left(\sqrt{2}\right)^2=2$, $2^2=4$

$\therefore$ Therefore, the value of $\sqrt{2}$ is greater than 1 and less than 2.

$\therefore$ $\sqrt{2}$ is not an integer.

$\therefore$ $\sqrt{2}$ is either a rational number or a irrational number. If $\sqrt{2}$ is a rational number let, $\sqrt{2}=\frac{p}{q}$; where p and q are natural numbers and co-prime to each other and $q>1$

or, $2=\frac{p^2}{q^2}$; squaring

or, $2q=\frac{p^2}{q}$; multiplying both sides by q.

Clearly $2q$ is an integer but $\frac{p^2}{q}$ is not an integer because p and q are co-prime natural numbers and $q>1$

$\therefore$ $2q$ and $\frac{p^2}{q}$ cannot be equal, i.e., $2q \neq \frac{p^2}{q}$

$\therefore$ Value of $\sqrt{2}$ cannot be equal to any number with the form $\frac{p}{q}$ i.e., $\sqrt{2} \neq \frac{p}{q}$

$\therefore$ $\sqrt{2}$ is an irrational number.

Example 2. Prove that, sum of adding of 1 with the product of four consecutive natural numbers becomes a perfect square number.

Solution : Let four consecutive natural numbers be $x,\ x+1,\ x+2,\ x+3$ respectively.

By adding 1 with their product we get,

$$x\,(x+1)(x+2)(x+3)+1 = x(x+3)\,(x+1)\,(x+2)+1$$
$$=(x^2+3x)(x^2+3x+2)+1$$
$$=a(a+2)+1; \qquad [x^2+3x=a]$$
$$=a(a+2)+1\ ;$$
$$=a^2+2a+1 \ =(a+1)^2 \ =(x^2+3x+1)^2;$$

which is a perfect square number.

$\therefore$ If we add 1 with the product of four consecutive numbers, we get a perfect square number.

Activity : Proof that, $\sqrt{3}$ is an irrational number

Classification of Decimal Fractions

Each real number can be expressed in the form of a decimal fraction.

For example, $2 = 2\cdot 0$, $\frac{2}{5} = 0.4$, $\frac{1}{3} = 0.333....$ etc. There are three types of decimal fractions :terminating decimals, recurring decimals and non-terminating decimals.

Terminating decimals : In terminating decimals, the finite numbers of digits are in the right side of a decimal points. For example, $0.12, 1.023, 7.832, 54.67, ...$ etc. are terminating decimals.

Recurring decimals : In recurring decimals, the digits or the part of the digits in the right side of the decimal points will occar repeatedly. For example, $3.333.....$, $2.454545......$, $5.12765765..........$ etc. are recurring decimals.

Non-terminating decimals : In non-terminating decimals, the digits in the right side of a decimal point never terminate, i.e., the number of digits in the right side of decimal point will not be finite neither will the part occur repeatedly. For example. $1.4142135......$, $2.8284271.......$ etc. are non-terminating decimals.

Terminating decimals and recurring decimals are rational numbers and non-terminating decimals are irrational numbers. The value of an irrational number can be determined upto the required number after the decimal point. If the numerator and denominator of a fraction can be expressed in natural numbers, that fraction is a rational number.

Activity : Classify the decimal fractions stating reasons :
1.723, 5.2333........, 0.0025, 2.1356124......., 0.0105105........ and 0.450123.......

Recurring decimal fraction :

Expressing the fraction $\frac{23}{6}$ into decimal fractions, we get,

$$\frac{23}{6} = 6\,)\,23\,(3\cdot833$$

$$\begin{array}{l}
18 \\ \hline
\ \ 50 \\
\ \ 48 \\ \hline
\ \ \ \ 20 \\
\ \ \ \ 18 \\ \hline
\ \ \ \ \ \ 20 \\
\ \ \ \ \ \ 18 \\ \hline
\ \ \ \ \ \ \ \ 20 \\
\ \ \ \ \ \ \ \ 18 \\ \hline
\ \ \ \ \ \ \ \ \ \ 2
\end{array}$$

Observe : It is found that, the process of division is not ended at the time of dividing a numerator of a fraction by its denominator. To convert it into decimal fraction in the quotient 3’occurs repeatedly. Here, $3.8333.....$ is a recurring decimal fraction.

If digit or successive digit of a decimal fractions a in the right side of the decimal point appear again and again, these are called recurring decimal fractions.

If a digit or successive digits of decimal fractions.

In recurring decimal fractions, the portion which occurs again and again, is called recurring part. In recurring decimal fraction, if one digit recurs, the recurring point is used upon it and if more than one digits recurs, the recurring point is used only upon the first and the last digits.

As for example : $2.555.......$ is written as $2.\dot{5}$ and $3.124124124........$ is written as $3.\dot{1}2\dot{4}$.

In recurring decimal fractions, if there is no other digit except recurring one, after decimal point it is called pure recurring decimal and if there is one digit or more after decimal point in addition to recurring one, it is called mixed recurring decimal. For example, $1.\dot{3}$ is a pure recurring decimal and $4.235\dot{1}\dot{2}$ is a mixed recurring decimal.

If there exists prime factors other than 2 and 5 in the denominator of the fraction, the numerator will not be fully divisible by denominator. As the last digit of successive divisions cannot be other than $1, 2, ..., 9$, at one stage the same number will be repeated in the remainder. The number in the recurring part is always smaller than that of the denominator.

Example 3. Express $\frac{3}{11}$ into decimal fraction.

Solution :

$$
\begin{array}{l}
11)30\,(0.2727 \\
\quad\underline{22} \\
\quad\;\;80 \\
\quad\;\;\underline{77} \\
\quad\;\;\;\;30 \\
\quad\;\;\;\;\underline{22} \\
\quad\;\;\;\;\;\;80 \\
\quad\;\;\;\;\;\;\underline{77} \\
\quad\;\;\;\;\;\;\;\;3
\end{array}
$$

Example 4. Express $\frac{95}{37}$ into decimal fraction.

Solution :

$$
\begin{array}{l}
37\,)95\,(2.56756 \\
\quad\;\underline{74} \\
\quad\;210 \\
\quad\;\underline{185} \\
\quad\;\;\;250 \\
\quad\;\;\;\underline{222} \\
\quad\;\;\;\;\;280 \\
\quad\;\;\;\;\;\underline{259} \\
\quad\;\;\;\;\;\;\;210 \\
\quad\;\;\;\;\;\;\;\underline{185} \\
\quad\;\;\;\;\;\;\;\;\;250 \\
\quad\;\;\;\;\;\;\;\;\;\underline{222} \\
\quad\;\;\;\;\;\;\;\;\;\;\;28
\end{array}
$$

Required decimal fraction = $0.2727......$
$= 0.\dot{2}\dot{7}$

Required decimal fraction =
$2.56756...... = 2.\dot{5}6\dot{7}$

Conversion of Recurring Decimal into Simple Fraction

Determining the value of recurring fraction :

Example 5. Express $0.\dot{3}$ into simple fraction.

Solution : $0.\dot{3} = 0.3333........$

$0.\dot{3} \times 10 = 0.333....... \times 10 = 3.333......$

and $0.\dot{3} \times 1 \quad = 0.333....... \times 1 \quad = 0.333......$

subtracting, $0.\dot{3} \times 10 - 0.\dot{3} \times 1 \qquad = 3$

or, $0.\dot{3} \times (10 - 1) = 3$ or $0.\dot{3} \times 9 = 3$

Therefore, $0.\dot{3} = \frac{3}{9} = \frac{1}{3}$

Required fraction is $\frac{1}{3}$

Example 6. Express $0.\dot{2}\dot{4}$ into simple fraction.

Solution : $0.\dot{2}\dot{4} = 0 \cdot 24242424........$

So, $0.\dot{2}\dot{4} \times 100 = 0.242424....... \times 100 = 24.2424......$

and $0.\dot{2}\dot{4} \times 1 \qquad = 0.242424....... \times 1 \qquad = 0.242424......$

Subtracting $0.\dot{2}\dot{4}(100 - 1) = 24$

or, $0.\dot{2}\dot{4} \times 9 \ = 24$ or, $0.\dot{2}\dot{4} = \frac{24}{9} = \frac{8}{33}$

Required fraction is $\frac{8}{33}$

Example 7. Express $5.1\dot{3}4\dot{5}$ into simple fraction.

Solution : $5.1\dot{3}4\dot{5} = 5.1345345345.........$

So, $5.1\dot{3}4\dot{5} \times 10000 = 5.1345345...... \times 10000 = 51345.345......$

and $5.1\dot{3}4\dot{5} \times 10 \qquad = 5.1345345....... \times 10 \quad = 51.345......$

Subtracting, $5.1\dot{3}4\dot{5} \times 9990 \qquad = 51345 - 51$

So, $5.1\dot{3}4\dot{5} = \frac{51345 - 51}{9990} = \frac{51294}{9990} = \frac{8549}{1665} = 5\frac{224}{1665}$

Required fraction is $5\frac{224}{1665}$

Example 8. Express $42.34\dot{7}\dot{8}$ into simple fraction.

Solution : $42.34\dot{7}\dot{8} = 42.347878........$

So, $42.34\dot{7}\dot{8} \times 10000 = 42.347878.........\times 10000 = 42348.7878$

and $42.34\dot{7}\dot{8} \times 100 = 42.347878........ \times 100 = 4234.7878$

Subtracting, $42.34\dot{7}\dot{8} \times 9900 = 423478 - 4234$

Therefore, $42.34\dot{7}\dot{8} = \frac{423478-4234}{9900} = \frac{419244}{9900} = \frac{34937}{825} = 42\frac{287}{825}$

Required fraction is $42\frac{287}{825}$

Explanation : From the examples 5, 6, 7 and 8 , it appears that,

- The recurring decimal has been multiplied by the number formed by putting at the right side of 1 the number of zeros equal to the number of digits in the right side of decimal point in the recurring decimal.
- The recurring decimal has been multiplied by the number formed by putting at the right side of 1 the number of zeros equal to the number of digits which are non-recurring after decimal point of the recurring decimal.
- the second product has been subtracted from the first product. By subtracting the second product from the first product the whole number has been obtained at the right side. Here it is observed that, the number of non-recurring part has been subtracted from the number obtained by removing the decimal and recurring points of recurring decimal fraction.
- The result of subtraction has been divided by the number formed by writing the same number of 9's equal to the number of digits of recurring part at the left and number of zeros equal to the number of digits of non-recurring part at the right.
- In the recurring decimals, converting into fractions the denominator is the number of 9's equal to the number of digits in the rec urring part and in right side of all 9's number of zeros equal to the number of digits in the non-recurring part. And the numerator in the result that is obtained by subtracting the number of the digits formed by omitting the digits of recurring part from the number formed by removing the decimal and recurring points of recurring decimal.

Remark : Any recurring decimal can also be converted into a fraction. All recurring decimals are rational numbers.

Example 9. Express $5.23\dot{4}5\dot{7}$ into simple fraction.

Solution : $5.23\dot{4}5\dot{7} = 5.23457457457.........$

So, $5.23\dot{4}5\dot{7} \times 100000 = 523457.457457$

and $5.23\dot{4}5\dot{7} \times 100 = 523.457457$

Subtracting, $5.23\dot{4}5\dot{7} \times 99900 = 522934$

Therefore, $5.23\dot{4}5\dot{7} = \dfrac{522934}{99900} = \dfrac{261467}{49950}$

Required fraction is $\dfrac{261467}{49950}$

Explanation : Here in the decimal part the recurring decimal has been multiplied first by 100000 (5 zeros at the right side of 1) as there are two digits at the left side of recurring part in the decimal portion, the recurring decimal has been multiplied by 100 (two zeros at the right side of 1) The second product has been subtracted from the first product. In one side of the result of subtraction is a whole number and at the other side of the result is $(100000-100) = 99900$ times of the value of the given recurring decimal. Dividing both the sides by 99900, the required fraction is obtained.

Activity : Express $0.\dot{4}\dot{1}$ and $3.04\dot{6}2\dot{3}$ into fractions.

Rules of Transformation of Recurring Decimals into Simple Fractions

Numerator of the required fraction = the result by subtracting the number obtained from exempting the decimal point of the given decimal point and the non-recurring part.

Denominator of the required fraction = Numbers formed by putting the number of 9 equal to the number of digits in the recurring part of the from the number of zeros equal to the number of digits in the non-recurring part. Here the above rules are directly applied to convert some recurring decimals into simple factions.

Example 10. Express $45.2\dot{3}4\dot{6}$ into simple fraction.

Solution : $45.2\dot{3}4\dot{6} = \dfrac{452346-452}{9990} = \dfrac{451894}{9990} = \dfrac{225947}{4995} = 45\dfrac{1172}{4995}$

Required fraction is $45\dfrac{1172}{4995}$

Example 11. Express $32.\dot{5}6\dot{7}$ into simple fraction.

Solution : $32.\dot{5}6\dot{7} = \dfrac{32567-32}{999} = \dfrac{32535}{999} = \dfrac{3615}{111} = \dfrac{1205}{37} = 32\dfrac{21}{37}$

Required fraction is $32\dfrac{21}{37}$.

Activity : Express $0.0\dot{1}\dot{2}$ and $3.31\dot{2}\dot{4}$ into fraction.

Similar recurring decimals and Non-similar Recurring decimals :

If the numbers of digits in non-recurring part of recurring decimals are equal and also numbers of digits in the recurring parts are equal, those are called similar recurring decimals. Other recurring decimals are called non-similar recurring decimals. For example : $12.\dot{4}\dot{5}$ and $6.\dot{3}\dot{2}$; $9.45\dot{3}$ and $125.89\dot{7}$ are similar recurring decimals. Again, $0.3\dot{4}5\dot{6}$ and $7.45\dot{7}8\dot{9}$; $6.4\dot{3}5\dot{7}$ and $2.89\dot{3}4\dot{5}$ are none-similar recurring decimals.

The Rules of Changing Non-Similar Recurring Decimals into Similar Recurring Decimals

The value of any recurring decimals is not changed, if the digits of its recurring part are written again and again, For Example, $6.45\dot{3}\dot{7} = 6.45\dot{3}73\dot{7} = 6.45\dot{3}7\dot{3} = 6.4537\dot{3}\dot{7}$. Here each one is a recurring decimal, $6.45373737.........$ is a non-terminating decimal.

It will be seen that each recurring decimal if converted into a simple fraction has the same value.

$$6.45\dot{3}\dot{7} = \frac{64537-645}{9900} = \frac{63892}{9900}$$

$$6.45\dot{3}73\dot{7} = \frac{6453737-645}{9900} = \frac{6453092}{9900} = \frac{63892}{9900}$$

$$6.4537\dot{3}\dot{7} = \frac{6453737-64537}{990000} = \frac{6389200}{990000} = \frac{63892}{9900}$$

In order to make the recurring decimals similar, number of digits in the non-recurring part of each recurring decimal is to be made equal to the number of digits of non-recurring part of that recurring decimal in which greatest number of digits in the non-recurring part exists and the number of digits in the recurring part of each recurring decimal is also to be made equal to the lowest common multiple of the numbers of digits of recurring parts of recurring decimals.

Example 12. Convert $5.\dot{6}$, $7.3\dot{4}\dot{5}$ and $10 \cdot 78\dot{4}2\dot{3}$ into similar recurring decimals.

Solution : The number of digits of non-recurring part of $5 \cdot \dot{6}$, $7 \cdot 3\dot{4}\dot{5}$ and $10.78\dot{4}2\dot{3}$ are $0, 1$ and 2 respectively. Here the number of dig its in the non-recurring part occurs in $10.78\dot{4}2\dot{3}$ and that number is 2. Therefore to make the recurring decimals similar the number of digits in the non-recurring part of each recurring decimal is to be made 2. Again, the numbers of digits to recurring parts of $5.\dot{6}$, $7.3\dot{4}\dot{5}$ and $10.78\dot{4}2\dot{3}$ are $1, 2$ and 3 respectively. The lowest common multiple of $1, 2$ and 3 is 6. So the number of digits in the recurring part of each recurring decimal would be 6 in order to make them similar.

So, $5.\dot{6} = 5.66\dot{6}6666\dot{6}$, $7.3\dot{4}\dot{5} = 7.34\dot{5}4545\dot{4}$ and $10.78\dot{4}2\dot{3} = 10.78\dot{4}2342\dot{3}$

Required similar recurring decimals are $5.66\dot{6}6666\dot{6}$, $7.34\dot{5}4545\dot{4}$, $10.78\dot{4}2342\dot{3}$ respectively.

Example 13. Convert 1.7643, $3.\dot{2}\dot{4}$ and $2.78\dot{3}4\dot{6}$ into similar recurring decimals.

Solution : In 1.7643 the number of digits in the non-recurring part means 4 digits after decimal point and here there is no recurring part.

In $3.\dot{2}\dot{4}$ the number of digits in the recurring and non-recurring parts are respectively 0 and 2.

In $2.78\dot{3}4\dot{6}$ the number of digits in the recurring and non-recurring parts are respectively 2 and 3.

The highest number of digits in the nonrecurring parts is 4 and the L.C.M. of the numbers of digits in the recurring parts i.e. 2 and 3 is 6. The numbers of digits is the recurring and nonrecurring parts of each decimal will be respectively 4 and 6.

$\therefore 1.7643 = 1.7643\dot{0}0000\dot{0}$; $3.\dot{2}\dot{4} = 3.2424\dot{2}4242\dot{4}$; $2.78\dot{3}4\dot{6} = 2.7834\dot{6}3463\dot{4}$

Required recurring similar decimals are $1.7643\dot{0}0000\dot{0}$, $3.2424\dot{2}4242\dot{4}$ and $2.7834\dot{6}3463\dot{4}$

Remark : In order to make the terminating fraction similar, the required number of zeros is placed after the digits at the extreme right of decimal point of each decimal fraction. The number of non-recurring decimals and the numbers of digits of non-recurring part of decimals after the decimal points are made equal using recurring digits. After non-recurring part the recurring part can be started from any digit.

Activity : Express 3.467, $2.01\dot{2}\dot{4}\dot{3}$ and $7.52\dot{5}\dot{6}$ into similar recurring fractions.

Addition and Subtraction of Recurring Decimals

In the process of addition or subtraction of recurring decimals, the recurring decimals are to be converted into similar recurring decimals. Then the process of addition or subtraction as that of terminating decimals is followed. If addition or subtraction of terminating decimals and recurring decimals together are done, in order to make recurring decimals similar, the number of digits of non-recurring part of each recurring should be equal to the number of digits between the numbers of digits after the decimal points of terminating decimals and that of the non-recurring parts of recurring decimals. The number of digits of recurring part of each recurring decimal will be equal to L.C.M. as obtained by applying the rules stated earlier and in case of terminating decimals, necessary numbers of zeros are to be used in its recurring parts. Then the same process of addition and subtraction is to be done following the rules of terminating decimals. The sum or the difference obtained in this way will not be the actual one. It should be

observed that in the process of addition of similar decimals if any number is to be carried over after adding the digits at the extreme left of the recurring part of the decimals then that number is added to the sum obtained and thus the actual sum is found. In case of subtraction the number to be carried over is to subtract from the difference obtained and thus actual result is found. The sum or difference which is found in this way is the required sum or difference.

Remark (a) : The sum or difference of recurring decimals is also a recurring decimal. In this sum or difference the number of digits in the non-recurring part will be equal to the number of digits in the non-recurring part of that recurring decimal, which have the highest number of digits in its non-recurring part. Similarly, the number of digits in the recurring part of the sum or the result of subtraction will be the equal to L.C.M. of the numbers of digits of recurring parts of recurring decimals. If there is any terminating decimals, the number of digits in the non-recurring part of each recurring decimal will be equal to the highest numbers of digits that occurs after the decimal point.

(b) Converting the recurring decimals into simple fractions, addition and subtraction may be done according to the rules as used in case of simple fractions and the sum or difference is converted into decimal fractions. But this process needs more time.

Example 14. Add : $3\cdot\dot{8}\dot{9}$, $2\cdot 1\dot{7}\dot{8}$ and $5\cdot 89\dot{7}9\dot{8}$

Solution : Here the number of digits in the non-recurring part will be 2 and the number of digits in the recurring part will be 6 which is L.C.M. of 2,2 and 3.

At first three recurring decimals are made similar.

$$
\begin{array}{lr}
3\cdot\dot{8}\dot{9} & = 3\cdot 89\dot{8}9898\dot{9} \\
2\cdot 1\dot{7}\dot{8} & = 2\cdot 127\dot{8}7878\dot{7} \\
\underline{5\cdot 89\dot{7}9\dot{8}} & \underline{= 589\dot{7}9879\dot{8}} \\
 & 11\cdot 97576574 \\
 & \underline{+2} \\
 & 11\cdot 97\dot{5}7657\dot{6}
\end{array}
$$

[$8+8+7+2=25$, Here 2 is the number to be carried over, 2 of 25 has been added.]

The required sum is $11\cdot 97\dot{5}7657\dot{6}$ or $11\cdot 97\dot{5}7\dot{6}$

Remark : In the sum the number in the recurring part is 575675. But the value is not changed if 576 is taken as the number of recurring part.

Note : To make clear the concept of adding 2 at the extreme right side, this addition is done in another method :

$$\begin{array}{lcl}
3\cdot\dot{8}\dot{9} & = & 3\cdot89\dot{8}9898\dot{9}\,|\,89 \\
2\cdot1\dot{7}\dot{8} & = & 2\cdot17\dot{8}7878\dot{7}\,|\,87 \\
5\cdot89\dot{7}9\dot{8} & = & 5\cdot89\dot{7}9879\dot{8}\,|\,79 \\
\hline
 & & 11.97\dot{5}7657\dot{6}\,|\,55
\end{array}$$

Here the number is extended upto 2 more digits after the completion of the recurring part. The additional digits are separated by drawing a vertical line. Then 2 has been carried over from the sum of the digits at the right side of the vertical line and this 2 is added to the sum of the digit at the left side of the vertical line. The digit in the right side of the vertical line are the same and the recurring point. Therefore both the sums are the same.

Example 15. Add : $8\cdot9\dot{4}7\dot{8}$, $2\cdot346$ and $4\cdot\dot{7}\dot{1}$.

Solution : To make the decimals similar, the number of digits of non-recurring parts would be 3 and that of recurring parts would be 6 which is L.C.M. of 3 and 2.

$$\begin{array}{lcr}
8\cdot9\dot{4}7\dot{8} & = & 8\cdot947\dot{8}4784\dot{7} \\
2\cdot346 & & 2\cdot346\dot{0}0000\dot{0} \\
4\cdot\dot{7}\dot{1} & = & 4\cdot717\dot{1}7171\dot{7} \\
\hline
 & & 16\cdot011019564 \\
 & & +1 \\
\hline
 & & 16\cdot011\dot{0}1956\dot{5}
\end{array}$$

[$8+0+1+1=10$, Here the digit in the second place on the left is 1 which is to be carried over. Therefore 1 of 10 is added.]

The required sum is $16\cdot011\dot{0}1956\dot{5}$.

Activity : Add :1. $2\cdot0\dot{9}\dot{7}$ and $5\cdot12\dot{7}6\dot{8}$ 2. $1\cdot34\dot{5}$, $0\cdot31\dot{5}7\dot{6}$ and $8\cdot056\dot{7}\dot{8}$

Example 16. Subtract $5\cdot24\dot{6}7\dot{3}$ from $8\cdot2\dot{4}\dot{3}$.

Solution : Here the number of digits in the non-recurring part would be 2 and that of recurring part is 6 which is L.C.M. of 2 and 3. Now making two decimal numbers similar, subtraction is done.

$$\begin{array}{lcr}
8\cdot2\dot{4}\dot{3} & = & 8\cdot24\dot{3}4343\dot{4} \\
5\cdot24\dot{6}7\dot{3} & = & 5\cdot24\dot{6}7367\dot{3} \\
\hline
 & & 2\cdot99669761 \\
 & & -1 \\
\hline
 & & 2\cdot99\dot{6}69760
\end{array}$$

[Subtracting 6 from 3, 1 is to be carried over.]

The required sum is $2\cdot99\dot{6}69760$.

Remark : If the digit at the beginning place of recurring point in the number from which deduction to be made is smaller than that of the digit in the number 1 is to be subtracted from the extreme right hand digit of the result of subtraction.

Note : In order to make the conception clear why 1 is subtracted, subtraction is done in another method as shown below :

$$
\begin{array}{lr}
8\cdot2\dot{4}\dot{3} & = 8\cdot24\dot{3}43434\dot{4}\,|\,34 \\
5\cdot24\dot{6}7\dot{3} & = 5\cdot24\dot{6}73673\dot{3}\,|\,67 \\
\hline
 & 2\cdot99\dot{6}69760\dot{0}\,|\,67
\end{array}
$$

The required difference is $2\cdot99\dot{6}69760\dot{0}\,|\,67$

Here both the differences are the same.

Example 17. Subtract $16\cdot\dot{4}3\dot{7}$ from $24\cdot45\dot{6}4\dot{5}$.

Solution :

$$
\begin{array}{lr}
24\cdot45\dot{6}4\dot{5} & = 24\cdot45\dot{6}4\dot{5} \\
16\cdot\dot{4}3\dot{7} & =16\cdot43\dot{7}4\dot{3} \\
\hline
 & 8\cdot01902 \\
 & -1 \\
\hline
 & 8\cdot01\dot{9}0\dot{1}
\end{array}
$$

[7 is subtracted from 6.1 is to be carried over.]

The required difference is $8.01\dot{9}0\dot{1}$

Note :

$$
\begin{array}{lr}
24\cdot45\dot{6}4\dot{5} & = 24\cdot45\dot{6}4\dot{5}\,|\,64 \\
16\cdot\dot{4}3\dot{7} & =16\cdot43\dot{7}4\dot{3}\,|\,74 \\
\hline
 & 8\cdot01\dot{9}0\dot{1}\,|\,90
\end{array}
$$

Activity : Subtract :1. 10 ·418 from $13\cdot12\dot{7}8\dot{4}$ 2. $9\cdot12\dot{6}4\dot{5}$ from $23\cdot03\dot{9}\dot{4}$

Multiplication and Division of Recurring Decimals :

Converting recurring decimals into simple fraction and completing the process of their multiplication or division, the simple fraction thus obtained when expressed into a decimal fraction will be the product or quotient of the recurring decimals. In the process of multiplication or division amongst terminating and recurring decimals the same method is to be applied. But in case of making division easier if both the divident and the divisior are of recurring decimals, we should convert them into similar recurring decimals.

Example 18. Multiply $4\cdot\dot{3}$ by $5\cdot\dot{7}$.

Solution : $4\cdot\dot{3} = \dfrac{43-4}{9} = \dfrac{39}{9} = \dfrac{13}{3}$

$$5\cdot\dot{7} = \frac{57-5}{9} = \frac{52}{9}$$

$$\therefore\ 4\cdot\dot{3} \times 5\cdot\dot{7} = \frac{13}{3}\times\frac{52}{9} = \frac{676}{27} = 25\cdot\dot{0}3\dot{7}$$

The required product is $25\cdot\dot{0}3\dot{7}$

Example 19. Multiply $0\cdot2\dot{8}$ by $42\cdot\dot{1}\dot{8}$.

Solution : $0\cdot2\dot{8} = \dfrac{28-2}{90} = \dfrac{26}{90} = \dfrac{13}{45}$

$$42\cdot\dot{1}\dot{8} = \frac{4218-42}{99} = \frac{4176}{99} = \frac{464}{11}$$

$$= \frac{13}{45}\times\frac{464}{11} = \frac{6032}{495} = 12\cdot1\dot{8}\dot{5}$$

The required product is $12\cdot1\dot{8}\dot{5}$

Example 20. $2\cdot5\times4\cdot3\dot{5}\times1\cdot2\dot{3}\dot{4} = \ ?$

Solution : $2\cdot5 = \dfrac{25}{10} = \dfrac{5}{2}$

$$4\cdot3\dot{5} = \frac{435-43}{90} = \frac{392}{90}$$

$$1\cdot2\dot{3}\dot{4} = \frac{1234-12}{990} = \frac{1222}{990} = \frac{611}{495}$$

$$\frac{5}{2}\times\frac{392}{90}\times\frac{611}{495} = \frac{196\times611}{8910} = \frac{119756}{8910} = 13\cdot44062....$$

The required product is $13\cdot44062$

Activity : 1. Multiply $1\cdot1\dot{3}$ by $2\cdot6$. 2. $0\cdot\dot{2}\times1\cdot\dot{1}\dot{2}\times0\cdot0\dot{8}\dot{1} = \ ?$

Example 21. Divide $7\cdot\dot{3}\dot{2}$ by $0\cdot2\dot{7}$.

Solution : $7\cdot\dot{3}\dot{2} = \dfrac{732-7}{99} = \dfrac{725}{99}$

$$0\cdot2\dot{7} = \frac{27-2}{90} = \frac{25}{90} = \frac{5}{18}$$

$$\therefore\ 7\cdot\dot{3}\dot{2} \div 0\cdot2\dot{7} = \frac{725}{99}\div\frac{5}{18} = \frac{725}{99}\times\frac{18}{5} = \frac{290}{11} = 26\cdot\dot{3}\dot{6}$$

The required quotient is $26 \cdot \dot{3}\dot{6}$

Example 22. Divide $2 \cdot \dot{2}71\dot{8}$ by $1 \cdot 9\dot{1}\dot{2}$

Solution : $2 \cdot \dot{2}71\dot{8} = \dfrac{22718 - 2}{9999} = \dfrac{22176}{9999}$

$$1 \cdot 9\dot{1}\dot{2} = \frac{1912 - 19}{990} = \frac{1893}{990}$$

$$\therefore 2 \cdot \dot{2}71\dot{8} \div 1 \cdot 9\dot{1}\dot{2} = \frac{22716}{9999} \div \frac{1893}{990} = \frac{22716}{9999} \times \frac{990}{1893} = \frac{120}{101} = 1 \cdot \dot{1}88\dot{1}$$

The required quotient is $1 \cdot \dot{1}88\dot{1}$

Example 23. Divide $9 \cdot 45$ by $2 \cdot 8\dot{6}\dot{3}$.

Solution : $9 \cdot 45 \div 2 \cdot 8\dot{6}\dot{3} = \dfrac{945}{100} \div \dfrac{2863 - 28}{990} = \dfrac{945}{100} \times \dfrac{990}{2835}$

$$= \frac{189 \times 99}{2 \times 2835} = \frac{33}{10} = 3 \cdot 3$$

The required quotient is : $3 \cdot 3$

Remark : Product of recurring decimals and quotient of recurring decimals may be or may not be a recurring decimal.

Activity : 1. Divide $0 \cdot \dot{6}$ by $0 \cdot \dot{9}$. 2. Divide $0 \cdot 7\dot{3}\dot{2}$ by $0 \cdot 0\dot{2}\dot{7}$

Non Terminating Decimals

There are many decimal fractions in which the number of digits after its decimal point is unlimited, again one or more than one digit does not occur repeatedly. Such decimal fractions are called non-terminating decimal fractions, For example, 5.134248513942307 .... is a non-terminating decimal number. The square root of 2 is a non terminating decimal. Now we want to find the square root of 2.

1	2	1·4142135........
	1	
24	100	
	96	
281	400	
	281	
2824	11900	
	11296	

$$\begin{array}{r|l}
28282 & 60400 \\
 & 56564 \\ \hline
282841 & 383600 \\
 & 282841 \\ \hline
2828423 & 10075900 \\
 & 8485269 \\ \hline
28284265 & 159063100 \\
 & 141421325 \\ \hline
 & 17641775
\end{array}$$

If the above process is continued for ever, it will never end.

$\therefore\ \sqrt{2} = 1 \cdot 4142135......$ is a non terminating decimal number.

The value upto the definite number of decimal places and the approximate value upto some decimal places.

It is not the same to find the value of nonterminating decimals upto definite number of decimal place and the approximate value into some decimal places. For example, $5 \cdot 4325893......$ upto four decimal places will be $5 \cdot 4325,$ but the approximate value of the decimal, $5 \cdot 4325893....$ upto four decimal places will be $5 \cdot 4326$. Here, the value upto 2 decimal places and the approximate value upto 2 decimal places are the same. This value is $5 \cdot 43$. In this way the approximate terminating decimals can also be found.

Remark : When it is needed to find the value upto some decimal places, the digits that occur in those places are to be written without any alternatives of those digits. If approximate values are to be identified, we should add 1 with the last digit when there is 5, 6, 7, 8 or 9 after the decimal places. But if it is 1, 2, 3 or 4 digits remain unchanged. In this case, correct value upto decimal place or approximate value upto decimal place are almost equal. We should find the value upto 1 place more to the required wanted value.

Example 24. Find the square root of 13 and write down the approximate value upto 3 decimal places.

Solution : $3\ )\ 13\ (3 \cdot 605551........$

$$\begin{array}{r|l}
 & 9 \\ \hline
66 & 400 \\
 & 396 \\ \hline
\end{array}$$

7205	40000
	36025
72105	3697500
	3605525
7211101	9197500
	7211101
	1986399

$\therefore$ The required square root is $3 \cdot 605551........$.

$\therefore$ The required approximate value upto 3 decimal places is $3 \cdot 606$.

Example 25. Find the value and approximate value of $4 \cdot 4623845.......$ upto $1, 2, 3, 4$ and 5 decimal places.

Solution : The value of $4 \cdot 4623845$ upto 1 decimal place is $4 \cdot 4$

and approximate value 1 ,, ,, ,, $4 \cdot 5$

Value upto 2 ,, ,, ,, $4 \cdot 46$

and approximate value upto 2 ,, ,, ,, $4 \cdot 46$

Value upto 3 ,, ,, ,, $4 \cdot 462$

and approximate value upto 3 ,, ,, ,, $4 \cdot 462$

Value upto 4 ,, $4 \cdot 4623$

and approximate value upto 4 ,, ,, ,, $4 \cdot 4624$

Value upto 5 ,, ,, ,, $4 \cdot 46238$

and approximate value upto 5 ,, ,, ,, $4 \cdot 46238$

Activity : Find the square root of 29 and find the value upto two decimal places and the approximate value upto two decimal place.

Exercise 1

1. Prove that, (a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{10}$ is an irrational number.
2. (a) Find the two irrational numbers between 0.31 and 0.12.

   (b) Find a rational and an irrational numbers between $\frac{1}{\sqrt{2}}$ and $\sqrt{2}$.
3. (a) Prove that, square of any odd integer number is an odd number.

   (b) Prove that, the product of two consecutive even numbers is divisible by 8.

4. Express into recurring decimal fractions :(a) $\frac{1}{6}$ (b) $\frac{7}{11}$ (c) $3\frac{2}{9}$ (d) $3\frac{8}{15}$

5. Express into simple fractions :

(a) $0\cdot\dot{2}$ (b) $0\cdot\dot{3}\dot{5}$ (c) $0\cdot 1\dot{3}$ (d) $3\cdot 7\dot{8}$ (e) $6\cdot 2\dot{3}0\dot{9}$

6. Express into similar recurring fractions :

(a) $2\cdot\dot{3}$, $5\cdot 2\dot{3}\dot{5}$ (b) $7\cdot 2\dot{6}$, $4\cdot 23\dot{7}$

(c) $5\cdot\dot{7}$, $8\cdot\dot{3}\dot{4}$, $6\cdot\dot{2}4\dot{5}$ (d) $12\cdot 32$, $2\cdot 1\dot{9}$, $4\cdot 32\dot{5}\dot{6}$

7. Add :(a) $0\cdot 4\dot{5}+0\cdot 13\dot{4}$ (b) $2\cdot 0\dot{5}+8\cdot 0\dot{4}+7\cdot 018$ (c) $0\cdot 00\dot{6}+0\cdot\dot{9}\dot{2}+0\cdot 0\dot{1}3\dot{4}$

8. Subtract :

(a) $3\cdot\dot{4}-2\cdot 1\dot{3}$ (b) $5\cdot\dot{1}\dot{2}-3\cdot 4\dot{5}$

(c) $8\cdot 49-5\cdot 3\dot{5}\dot{6}$ (d) $19\cdot 34\dot{5}-13\cdot 2\dot{3}4\dot{9}$

9. Multiply:

(a) $0\cdot\dot{3}\times 0\cdot\dot{6}$ (b) $2\cdot\dot{4}\times 0\cdot\dot{8}\dot{1}$ (c) $0\cdot 6\dot{2}\times 0\cdot\dot{3}$ (d) $42\cdot\dot{1}\dot{8}\times 0\cdot 2\dot{8}$

10. Divide :

(a) $0\cdot\dot{3}\div 0\cdot\dot{6}$ (b) $0\cdot 3\dot{5}\div 1\cdot\dot{7}$ (c) $2\cdot 3\dot{7}\div 0\cdot 4\dot{5}$ (d) $1\cdot\dot{1}8\dot{5}\div 0\cdot\dot{2}\dot{4}$

11. Find the root (upto three decimal places) and write down the approximate values of the square roots upto two decimal places :

(a) 12 (b) $0\cdot\dot{2}\dot{5}$ (c) $1\cdot 3\dot{4}$ (d) $5\cdot 1\dot{3}0\dot{2}$

12. Find the rational and irrational numbers from the following numbers :

(a) $0\cdot\dot{4}$ (b) $\sqrt{9}$ (c) $\sqrt{11}$ (d) $\frac{\sqrt{6}}{3}$ (e) $\frac{\sqrt{8}}{\sqrt{7}}$ (f) $\frac{\sqrt{27}}{\sqrt{48}}$ (g) $\frac{\frac{2}{3}}{\frac{3}{7}}$ (h) $5\cdot\dot{6}3\dot{9}$

13. Simplify :

(a) $(0\cdot\dot{3}\times 0\cdot 8\dot{3})\div(0\cdot 5\times 0\cdot\dot{1})+0\cdot 3\dot{5}\div 0\cdot 0\dot{8}$

(b) $[(6\cdot 27\times 0\cdot 5)\div\{(0\cdot 5 \times 0\cdot 75)\times 8\cdot 36\}]$
$\div\{(0\cdot 25 \times 0\cdot 1)\times(0\cdot 75 \times 21\cdot\dot{3})\times 0\cdot 5\}$

14. $\sqrt{5}$ and 4 are two real numbers.

(a) Which one is rational and which one is irrational.

(b) Find the two irrational numbers between $\sqrt{5}$ and 4.

(c) Pove That, $\sqrt{5}$ is an irrational number.